Saturation and cavity-loss optimization in free-electron lasers

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In this paper we reconsider the problem of the optimum cavity losses for the free-electron-laser (FEL) operation. We use the results of a recently proposed one-dimensional saturation model and show that for large intracavity intensities the FEL behaves like conventional laser systems.

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In a number of previous papers [1] a simple one-dimensional model to describe saturation effects in free-electron lasers (FEL's) has been developed [2]. One of the main results of the above quoted papers has been the derivation of two-gain versus intracavity-intensity scaling relations, namely,

$$G(I)=\frac{G_{\max}}{1+0.86(I/I_s)+0.14(I/I_s)^2} \tag{1a}$$

and

$$G(I)=G_{\max}\frac{1-e^{-(\pi/2)(I/I_s)}}{(\pi/2)(I/I_s)}\ , \tag{1b}$$

where $G_{\max}$ is the maximum gain of the system and I_s is the saturation intensity. This last quantity plays the same role as in conventional laser physics [3] and is the value of the intensity halving the small signal gain of the device. In the hypothesis of a low-gain homogeneously bounded regime I_s is provided by the following practical formula:

$$I_s\left[\frac{\text{W}}{\text{cm}^2}\right]=6.9\times 10^8\left[\frac{\gamma}{N}\right]^4\frac{1}{(\lambda_u\ [\text{cm}]k[JJ])^2}\ , \tag{2}$$

with γ being the electron relativistic factor, and N, λ_u, and k being the undulator number of periods, the period length, and the parameter, respectively. Finally

$$[JJ]=J_0(\xi)-J_1(\xi)\ ,\quad \xi=\frac{1}{4}\frac{k^2}{1+k^2/2} \tag{3}$$

is the Bessel-factor correction.

In Fig. 1 we have plotted the gain versus $x=I/I_s$ along with the relative differences between Eqs. (1a) and (1b). It is evident that for $x\leq 3.8$ the agreement between the scaling relations is within 5% and for larger values it tends to increase. We must emphasize that Eq. (1b) correctly reproduces the numerical results for a larger interval of x values (we have checked up to $x=10$). We have therefore strong reasons to believe that, within the limits of the one-dimensional model, Eq. (1b) is an accurate description of the gain saturation versus intracavity intensity. Equation (1a) is, however, interesting for at least three reasons.

(1) It reproduces more than satisfactorily the numerical trend for a large range of x values.

(2) It is strongly reminiscent of the conventional laser-gain saturation formula [3].

(3) It is more manageable than Eq. (1b).

A quantity of paramount importance in oscillator FEL physics is the intracavity equilibrium intensity, which is defined as the value of the intensity for which the net gain of the system is zero, namely,

$$G(I_e)=\frac{\eta}{1-\eta}\ , \tag{4}$$

where η denotes the total cavity losses. The intensity I_e is immediately obtained from Eq. (1a) which yields

$$x_e=\frac{I_e}{I_s}\simeq\frac{1}{0.28}\left\{-0.86+\left[0.86^2+0.56\left[G_{\max}\frac{1-\eta}{\eta}-1\right]\right]^{1/2}\right\}. \tag{5a}$$

The derivation of x_e from (1b) requires more work and the analysis of numerical data indicates that

$$x_e\simeq\frac{2}{\pi}\frac{G_{\max}(1-\eta)}{\eta}\left\{1-\exp\left[-\frac{1.8}{(1+G_{\max})}\frac{G_{\max}(1-\eta)-\eta}{\eta}\right]\right\}, \tag{5b}$$

when η approaches zero. Equations (5a) and (5b) have strongly different behaviors (see Fig. 2), the first scales in fact as $1/\sqrt{\eta}$ and the second as $1/\eta$. Equation (5a) is not reliable for small η because the values of x in this region are large and out of the region of validity of Eq. (1a).

Assuming, furthermore, that the system is dominated by active losses, the output-coupled power will be given by

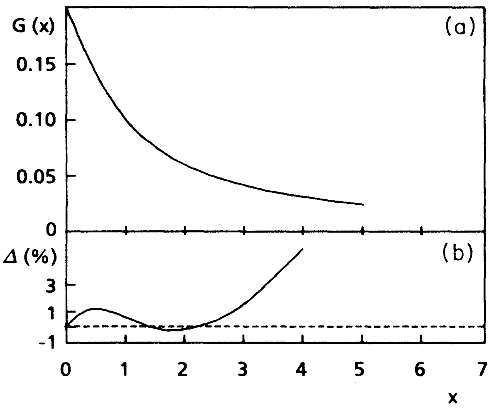

FIG. 1. (a) Gain vs $x=I/I_s$. The difference between the values predicted by Eqs. (1a) and (1b) are not appreciable on this scale, $G_{max}=0.2$. (b) $\Delta=(G_{(1b)}-G_{(1a)})/G_{(1b)}$ vs x. The parameter Δ provides the relative differences between the predictions of Eqs. (1a) and (1b).

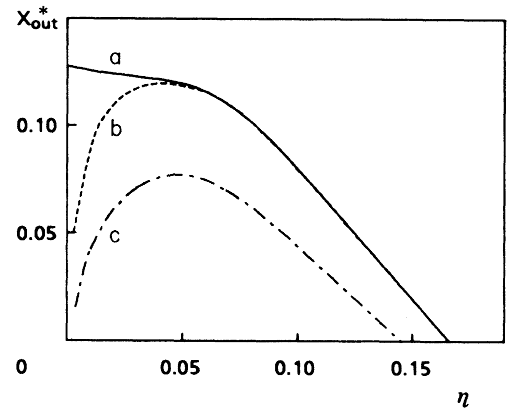

FIG. 3. Output coupled power $X^*_{out}=I^*_{out}/I_s$ vs the cavity losses, $G_{max}=0.2$. (a) Prediction from Eq. (5b). (b) Prediction from Eq. (5a). (c) Passive losses 0.02, prediction from Eq. (5b).

$$X^*_{out}=\eta X_e(\eta) \; . \tag{6}$$

The behavior of X^*_{out} vs η is shown in Fig. 3. The parametrization (5a) indicates the existence of an optimum η, namely,

$$\eta^*=G_{max}\frac{0.86\left[\dfrac{0.14}{1+G_{max}}\right]^{1/2}-0.28}{0.86^2-0.56(1+G_{max})} \tag{7}$$

and for small η, X^*_{out} approaches zero. In case (5b) X^*_{out} vs η coincides with the previous case for $\eta>\eta^*$, while for $\eta<\eta^*$ the output-coupled intensity is almost a constant, whose values are close to

$$X^*_{out}\simeq\frac{2}{\pi}G_{max} \; . \tag{8}$$

In analogy to conventional lasers, the cavity losses optimizing the output-coupled power are $\eta=0$; however, for FEL's operating in the low-gain regime, X^*_{out} is constant in the range $0<\eta<\eta^*$.

The losses η^* represent a kind of threshold optimum value in the sense that below it, the extracted power is almost flat and above it, X^*_{out} tends to decrease. Furthermore, noticing that small losses require longer times to reach equilibrium, the choice of η^* for optimum loss of the system might be certainly convenient from a practical point of view.

We must also emphasize that passive losses also play a role in the saturation process. Assuming therefore that

$$\eta=\eta_a+\eta_p \; , \tag{9}$$

with $\eta_{a,p}$ denoting active and passive losses, respectively, we get

$$X^*_{out}=\eta_a X_e(\eta_a+\eta_p) \; . \tag{10}$$

We show in Fig. 3(c) the behavior of X^*_{out} versus the active losses η_a, keeping fixed the passive losses η_p. It is interesting to notice that in this case a nonzero optimum value of η_a can be defined.

In this low-gain homogeneously broadened regime the maximum gain is linearly linked to the gain coefficient g_0 by the relation $G_{max}=0.85g_0$. In this hypothesis we get from Eq. (8)

$$I^*_{out}\simeq 0.5g_0 I_s \; , \tag{11}$$

which is the same as quoted in Ref. [1] and yielding an intrinsic efficiency of $1/4N$.

Before closing the paper it is worth clarifying the extension of the above analysis to the high-gain regime and to the inclusion of inhomogeneous broadening effects. Such an extension is almost straightforward since the functional forms of gain versus intensity saturation are still those given by Eqs. (1a) and (1b) with the same quoted limitations for the range of I/I_s. The only quantities which should be redefined in addition to the high-gain and inhomogeneous broadening contributions are I_s and G_{max}.

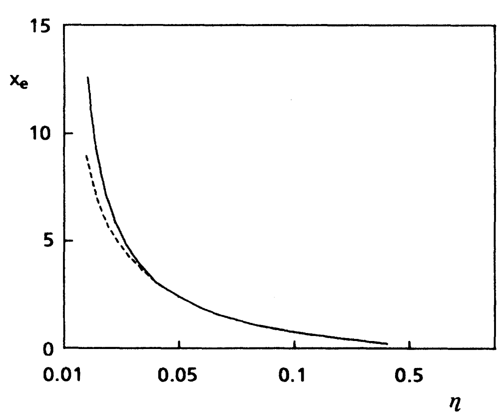

FIG. 2. Equilibrium intracavity intensity ($x_e=I_e/I_s$) vs the cavity losses η, $G_{max}=0.2$. Solid line, Eq. (5b); dashed line, Eq. (5a).

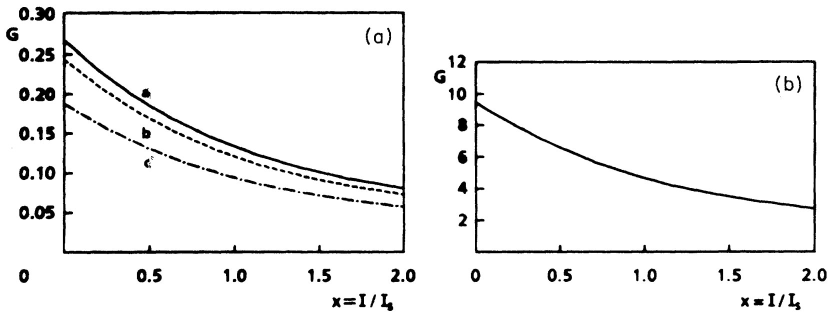

FIG. 4. (a) Gain vs $x=I/I_s$ ($g_0=0.3$, $N=30$). Curve a: $\sigma_\epsilon=0$, $I_s=293$ MW/cm^2. Curve b: $\sigma_\epsilon=2‰$, $I_s=334$ MW/cm^2. Curve c: $\sigma_\epsilon=4‰$, $I_s=436$ MW/cm^2. Numerical and analytical curves are indistinguishable. (b) Gain vs $x=I/I_s$ ($g_0=5$, $N=30$, $\sigma_\epsilon=0$, $I_s=180$ MW/cm^2).

Before specifying how I_s and $G_{\max}$ should be modified it is better to clarify what is meant by high gain and the inhomogeneously broadened regime. By high gain we mean that the small-signal gain coefficient g_0 is assumed to be larger than 0.3, and by inhomogeneous broadening we mean that the effect of the beam parameters (energy spread and emittances) are not negligible and may modify the gain value.

In Fig. 4 we show the gain versus I/I_s for a low-gain FEL, including inhomogeneous broadening, and for a high-gain FEL ($g_0=5$). The curves have been obtained by a numerical integration of the pendulum equation and the agreement with (1a) and (1b) is very good. It is, however, evident that when the inhomogeneous broadening, due to energy spread σ_ϵ is included [see Fig. 4(a)], I_s increases with an increasing energy spread, according to the relation [$I_s(0)$ is the value provided by Eq. (2)]

$$I_s(\mu_\epsilon)\simeq I_s(0)(1+c\mu_\epsilon^2)\ , \quad \mu_\epsilon=4N\sigma_\epsilon\ , \tag{12a}$$

where c is a constant around 2. A similar behavior has been observed when the emittance is included. When g_0 increases, see Fig. 4(b), I_s is a decreasing function of g_0 and in the case of $g_0\le 5$ can be reproduced by

$$I_s(g_0)\simeq\frac{I_s(0)}{(1+ag_0)}\ , \tag{12b}$$

where a is a constant around 0.12. The formulas allowing the parametrization of $G_{\max}$, including the high-gain and inhomogeneous broadening corrections, have been discussed in Ref. [4] and are not reported here for the sake of brevity. The above elements indicate that the model can be extended to the high-gain and inhomogeneously broadened regime without significant modifications.

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[3] See, e.g., A. Yariv, *Quantum Electronics* (Wiley, New York, 1975).
[4] G. Dattoli, H. Fang, L. Giannessi, M. Richetta, A. Torre, and R. Caloi, Nucl. Instrum. Methods A **285**, 108 (1989).